Simple linear equations

In this leaflet we look at the solution of simple linear equations in one variable - this means there will be no x^2 terms and no terms involving higher powers of x. There will be no functions of x such as $\sin x$, $\sqrt{x}$. For example, we will see how to solve the equation $3x + 15 = x + 25$. To **solve** such an equation means to find a value for x which makes the left- and right-hand sides equal.

Example

Suppose we wish to solve the equation

$$3x + 15 = x + 25$$

The important thing to remember about any equation is that the equals sign represents a balance. What an equals sign says is that what's on the left-hand side is exactly the same as what's on the right-hand side. So, if we do anything to one side of the equation we have to do it to the other side. If we do this, the balance is preserved. The first step in solving this equation is to perform operations on both sides so that terms involving x appear on one side only, usually the left. We can subtract x from each side, because this will remove it entirely from the right, and give

$$2x + 15 = 25$$

We can subtract 15 from both sides to give

$$2x = 10$$

and finally, by dividing both sides by 2 we obtain

$$x = 5$$

So the solution of the equation is $x = 5$. This solution should be checked by substitution into the original equation in order to check that both sides are the same. If we do this, the left is $3(5) + 15 = 30$. The right is $5 + 25 = 30$. So the left equals the right and the solution is correct.

Example

Suppose we wish to solve the equation $2x + 3 = 6 - (2x - 3)$.

We first remove the brackets on the right to give

$$2x + 3 = 6 - 2x + 3$$

so that

$$2x + 3 = 9 - 2x$$

This now has the same form as the equation in the first example. We can remove terms involving x from the right by adding $2x$ to each side.

$$4x + 3 = 9$$

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Now subtract 3 from each side:

$$4x = 6$$

so that

$$x = \frac{6}{4} \quad (\text{ by dividing both sides by 4})$$
$$= \frac{3}{2}$$

When solving an equation you may:

- add or subtract the same quantity to/from each side
- multiply or divide both sides by the same quantity
- apply other operations to the whole of both sides, e.g. square root both sides

Doing exactly the same to both sides preserves the balance in the original equation.

Example

Suppose we need to solve the equation

$$\frac{4(x+2)}{5} = 7 + \frac{5x}{13}$$

Here, the fractions are the cause of the difficulty. We try to remove them and work with whole numbers. Multiplying both sides by 5 and and then by 13 will remove the fractions. This is equivalent to multiplying both sides by the lowest common denominator, which is $5 \times 13 = 65$.

$$\frac{4(x+2)}{5} = 7 + \frac{5x}{13}$$
$$65 \times \frac{4(x+2)}{5} = 65 \times 7 + 65 \times \frac{5x}{13}$$
$$^{13}\cancel{65} \times \frac{4(x+2)}{\cancel{5}} = 65 \times 7 +^{5}\cancel{65} \times \frac{5x}{\cancel{13}}$$
$$52(x+2) = 455 + 25x$$

This is a much more familiar form, like the earlier examples. You should complete the solution yourself to obtain $x = 13$.

Exercises

Solve each of the following equations:

(a) $2 + 3x = 23$ (b) $3x + 14 = 2x - 5$ (c) $5(x - 4) = 2(x - 3)$ (d) $\frac{1}{2}(x + 7) = \frac{3x}{5} + 9$.

Answers

(a) $x = 7$ (b) $x = -19$ (c) $x = 14/3$ (d) $x = -55$.