What is a logarithm ?

Logarithms appear in many applications and familiarity with them is essential. They are used to write expressions involving powers in different forms.

Logarithms

Study the statement

$$100 = 10^2$$

In this statement we say that 10 is the **base** and 2 is the **power** or **index**. **Logarithms** provide an alternative way of writing a statement such as this. We rewrite it as

$$\log_{10} 100 = 2$$

This is read as 'log to the base 10 of 100 is 2'. These alternative forms are shown in Figure 1.

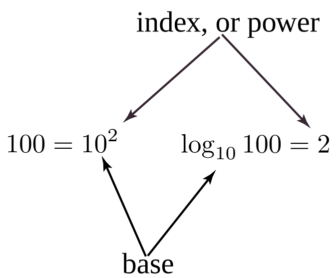

Figure 1. Note the positions of the different quantities in these two alternative forms.

As another example, since

$$2^5 = 32$$

we can write

$$\log_2 32 = 5$$

Here the base is 2 and the power is 5. We read this as 'log to the base 2 of 32 is 5'.

More generally,

$$\boxed{\text{if} \qquad a = b^c, \qquad \text{then} \qquad \log_b a = c}$$

Exercises

1. Rewrite the following expressions in logarithm form. Do not try to use a calculator.

(a) $3^2 = 9$ (b) $5^4 = 625$ (c) $10^3 = 1000$ (d) $10^{-2} = 0.01$

(e) $10^1 = 10$ (f) $2^1 = 2$ (g) $e^1 = e$ (h) $8^1 = 8$.

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2. Rewrite the following expressions in an equivalent form without using logarithms. Do not use a calculator.

(a) $\log_2 256 = 8$ (b) $\log_{10} 10000 = 4$ (c) $\log_4 64 = 3$ (d) $\log_{10} 0.1 = -1$

(e) $\log_3 3 = 1$ (f) $\log_9 9 = 1$ (g) $\log_8 1 = 0$ (h) $\log_2 1 = 0$.

Using a calculator to find logarithms

The only restriction that is placed on the value of the base is that it is a positive real number excluding the number 1. In practice logarithms are calculated using only a few common bases. Most frequently you will meet bases 10 and e. The letter e stands for the number 2.718... and is used because it is found to occur in the mathematical description of many physical phenomena. The number e is called the **exponential constant**. Your calculator will be able to calculate logarithms to bases 10 and e. Usually the 'log' button is used for base 10, and the 'ln' button is used for base e. ('ln' stands for 'natural logarithm'). Check that you can use your calculator correctly by verifying that

$$\log_{10} 73 = 1.8633 \text{ (to 4 decimal places)}$$

and

$$\log_e 5.64 = 1.7299 \text{ (to 4 decimal places)}$$

You may also like to verify the alternative forms

$$10^{1.8633} = 73 \qquad \text{and } e^{1.7299} = 5.64$$

Occasionally we need to find logarithms to other bases. For example, logarithms to the base 2 are used in communications engineering and information technology. Your calculator can still be used but we need to apply a formula for changing the base. This is dealt with on the leaflet *Logs - changing the base*.

Answers

1. (a) $\log_3 9 = 2$ (b) $\log_5 625 = 4$ (c) $\log_{10} 1000 = 3$ (d) $\log_{10} 0.01 = -2$

   (e) $\log_{10} 10 = 1$ (f) $\log_2 2 = 1$ (g) $\log_e e = 1$ (h) $\log_8 8 = 1$

2. (a) $2^8 = 256$ (b) $10^4 = 10000$ (c) $4^3 = 64$ (d) $10^{-1} = 0.1$

   (e) $3^1 = 3$ (f) $9^1 = 9$ (g) $8^0 = 1$ (h) $2^0 = 1$

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