Expanding or removing brackets

In this leaflet we see how to expand an expression containing brackets. By this we mean to rewrite the expression in an equivalent form without any brackets in.

Single brackets

If we have a number, or a single algebraic term, multiplying bracketed terms, then all terms in the brackets must be multiplied as shown in the following examples.

$$a(b+c) = ab + ac \qquad a(b-c) = ab - ac$$

Example

Expand $3(x+2)$.

The 3 outside must multiply both terms inside the brackets:

$$3(x+2) = 3x + 6$$

Example

Expand $-3a^2(3-b)$.

Both terms inside the brackets must be multiplied by $-3a^2$:

$$-3a^2(3-b) = -9a^2 + 3a^2b$$

Example

Expand $x(x-y)$.

The x outside must multiply both terms inside the brackets:

$$x(x-y) = x^2 - xy$$

Example

Expand $(x+5)x$.

Here, the brackets appear first, but the principle is the same. Both terms inside must be multiplied by the x outside:

$$(x+5)x = x^2 + 5x$$

Exercises

1. Remove the brackets from the following expressions.

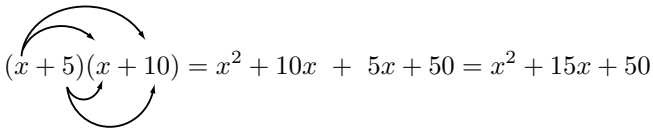
a)	$5(x+4)$	b)	$2(y-3)$	c)	$4(3-a)$	d)	$x(2+x)$	e)	$p(q+3)$
f)	$-3(2+a)$	g)	$s(t-s)$	h)	$-2(b-3)$	i)	$5a(2b+3c)$	j)	$-y(2x-5y)$

Multiplying together two bracketed terms

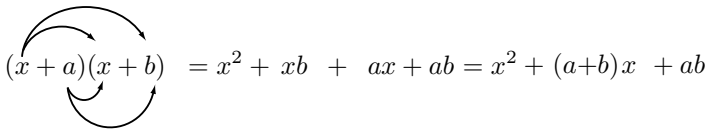
Supppose we want to multiply out expressions where there are two brackets multiplying each other, for example, $(x+5)(x+10)$. We imagine that the term $(x+5)$ is a single quantity and use it to multiply both the x and the 10 in the second pair of brackets:

$$\begin{aligned}(x+5)(x+10) &= (x+5)x \;+\; (x+5)10 \\ &= x^2 + 5x \;+\; 10x + 50 \\ &= x^2 + 15x + 50\end{aligned}$$

Having seen how to do this, we can shorten the process; to find $(x+5)(x+10)$ we must ensure that each term in the first bracket multiplies each term in the second. The arrows in the figure below help us to see that all terms have been taken into account:



More generally



Example

Expand $(x-7)(x-10)$.

$$\begin{aligned}(x-7)(x-10) &= x^2 - 10x - 7x + 70\\ &= x^2 - 17x + 70\end{aligned}$$

Example

Expand $(2x-3)(x+1)$.

$$\begin{aligned}(2x-3)(x+1) &= 2x^2 + 2x - 3x - 3\\ &= 2x^2 - x - 3\end{aligned}$$

Example

Expand $(x+6)(x-6)$.

$$\begin{aligned}(x+6)(x-6) &= x^2 - 6x + 6x - 36\\ &= x^2 - 36\end{aligned}$$

Example

Expand $(3x-2)(3x+2)$.

$$\begin{aligned}(3x-2)(3x+2) &= 9x^2 + 6x - 6x - 4\\ &= 9x^2 - 4\end{aligned}$$

Exercises

2. Expand each of the following.

a) $(x+2)(x+3)$ b) $(a+b)(c+3)$ c) $(y-3)(y+2)$ d) $(2x+1)(3x-2)$
e) $(3x-1)(3x+1)$ f) $(5x-1)(x-5)$ g) $(2p+3q)(5p-2q)$ h) $(x+2)(2x^2-x-1)$

Answers

1. a) $5x+20$ b) $2y-6$ c) $12-4a$ d) $2x+x^2$ e) $pq+3p$
f) $-6-3a$ g) $st-s^2$ h) $-2b+6$ i) $10ab+15ac$ j) $-2xy+5y^2$

2. a) x^2+5x+6 b) $ac+3a+bc+3b$ c) y^2-y-6 d) $6x^2-x-2$
e) $9x^2-1$ f) $5x^2-26x+5$ g) $10p^2+11pq-6q^2$ h) $2x^3+3x^2-3x-2$

