Factorising complete squares

The technique of factorising a quadratic expression has been explained on leaflet *Factorising quadratic expressions*. There is a special case of quadratic expression known as **a complete square**. This leaflet explains what this means and how such expressions are factorised.

What is meant by a complete square ?

A quadratic expression is called a **complete square** when it can be written in the form $(\quad)^2$, that is as a single term, squared.

Consider the following example.

Example

Factorise $x^2 + 10x + 25$.

We write

$$x^2 + 10x + 25 = (x \qquad)(x \qquad)$$

and seek two numbers which add to give 10 and multiply to give 25. The two required numbers are 5 and 5 and so

$$x^2 + 10x + 25 = (x + 5)(x + 5)$$

Because both brackets are the same the result can be written as $(x + 5)^2$. This is a single term, squared, - that is, a **complete square**.

Example

Factorise $x^2 - 8x + 16$.

Proceeding as before, we write

$$x^2 - 8x + 16 = (x \qquad)(x \qquad)$$

and seek two numbers which add to give -8 and multiply to give 16. The two required numbers are -4 and -4 and so

$$x^2 - 8x + 16 = (x - 4)(x - 4)$$

The result can be written as $(x - 4)^2$, a **complete square**.

More complicated examples can occur, for example when there is a number in front of the x^2. Work through the following example.

Example

Factorise $25x^2 - 20x + 4$.

Note that $25x^2$ can be written as $(5x)^2$, a squared term. Note also that $4 = 2^2$. In this case, by inspection,

$$25x^2 - 20x + 4 = (5x - 2)(5x - 2)$$

The result can be written as $(5x - 2)^2$, a **complete square**.

Do not worry if you have difficulty with this last example. The skill will come with practice.

Exercises

1. Factorise the following.

a)	$x^2 + 18x + 81$	b)	$x^2 - 4x + 4$	c)	$x^2 - 22x + 121$
d)	$25x^2 + 40x + 16$	e)	$64x^2 + 16x + 1$		

Answers

1.

a)	$(x + 9)^2$	b)	$(x - 2)^2$	c)	$(x - 11)^2$
d)	$(5x + 4)^2$	e)	$(8x + 1)^2$		

