**National University of Singapore**

**Department of Mathematics**

**Level 1000 (2005/2006) Semester 2 MA1102R Calculus Tutorial 10**

1. Find the following integrals.

a. $\int \frac{x^2}{\sqrt{25-x^2}}dx.$ b. $\int \frac{x^2}{\sqrt{9+x^2}}dx.$ c. $\int \frac{\sqrt{x^2-25}}{x}dx.$ d. $\int \frac{x^5+x^4+x^3+x^2+x+1}{x^3-x}dx.$

e. $\int \frac{x^2+3}{x(x^2+x+1)}dx.$ f. $\int \frac{x^2+3x+5}{(x^2+2x+5)^2}dx.$ g. $\int \sqrt{9-\sqrt{x}}\,dx.$

2. (i) Find the arc length of the curve $y^2 = x^3$ from (0,0) to (1,1).

(ii) Find the arc length of the curve $y = \ln(x)$ from $x = ½$ to $x = 2$.

(iii) The area bounded by $y = e^{-x}$ the axes, and the line $x = 2$ is revolved about the $x$-axis. Find the volume generated.

(iv) The area bounded by the parabola $y = \frac{x^2}{a}$ the $x$-axis, and the line $x = b$ is revolved about the $x$-axis. Find the volume generated.

(v) Find the areas of the surface obtained by revolving the part of the curve $y = \sqrt{x}$ that lies between $x = 2$ and $x = 6$ around the $x$ axis.

(vi) Find the areas of the surface obtained by revolving the part of the curve $y = x^3$ that lies between $x = 0$ and $x = 2/3$ around the $x$ axis.

3. Evaluate, if it exists, each of the following limits.

a. $\lim_{x\to\pi} \frac{\sin^2(2x)}{1+\cos(5x)}.$ b. $\lim_{x\to 0} \frac{2x-\ln(2x+1)}{1-\cos(3x)}.$ c. $\lim_{x\to 1}\left(\frac{1}{\ln(x)}-\frac{1}{x-1}\right).$

d. $\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{1}{e^{x-2}-1}\right).$ e. $\lim_{x\to 0} \frac{\int_0^x \sin(t^2+t)dt}{\tan(x^2)}.$ f. $\lim_{x\to 0} \frac{\tan^2(x^2)}{x^3}.$

g. $\lim_{x\to 0^+} x^{\sin(x^3)}.$ h. $\lim_{x\to 0} (e^x+7x)^{\frac{1}{x}}.$

4. Evaluate the following limits.

a. $\lim_{x\to+\infty} \frac{x^7}{e^x}.$ b. $\lim_{x\to+\infty} \frac{(\ln(x))^7}{x}.$ c. $\lim_{x\to 0^+} \tan(x^3)\ln(x).$ d. $\lim_{x\to 0^+} (\sin(x))^{x^3}.$

5. Use the Maclaurin polynomial for the function $f(x) = \ln(1+x)$ to compute the value of ln(1.2) accurate to three decimal places.

6. Find the following limits (i) $\lim_{n\to\infty} \frac{n}{n+1}$ (ii) $\lim_{n\to\infty} \frac{n+1}{n^3+4}.$

7. Use Squeeze Theorem or the Comparison test to prove

(i) $\lim_{n\to\infty} \frac{\sin(n)}{n} = 0$ (ii) $\lim_{n\to\infty} \frac{n!}{n^n} = 0$

8. Use Comparison test or otherwise determine the convergence of the following series

(i) $\sum_{1}^{\infty} \frac{1}{1+n^2}$ (ii) $\sum_{1}^{\infty} \frac{n+1}{n(n+2)}$ (iii) $\sum_{1}^{\infty} \frac{\ln(n)}{\sqrt{n+1}}$

9. Use the Ratio Test to determine the convergence of the following series.

(i) $\sum_{1}^{\infty} \frac{(n!)^2}{(2n)!}$ (ii) $\sum_{1}^{\infty} \frac{n!}{n^n}$ (iii) $\sum_{1}^{\infty} \frac{n^2}{2^n}$